Cooling trapped particles using feedback schemes

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Trapped particles are a fundamental tool for the implementation of quantum state control, engineering, and quantum information processing. The essential prerequisite is the ability to cool down the trapped particle to its motional ground state. This goal has been already achieved both with ions in rf-traps [1], and with neutral atoms in optical dipole traps [2], using resolved sideband cooling. It may happen however, that a trapped ion that is a favorable candidate for quantum information processing since it posseses a hyperfine structure with long coherence times (as for example $^{25}\mathrm{Mg}^{+}$), is not suitable for resolved sideband cooling. In such cases it may be helpful to have an alternative cooling technique, which can be applied when resolved sideband cooling is impractical to use. Here we present a new scheme based on feedback, able to achieve motional ground state cooling. The basic idea is to realize an effective and continuous measurement of the position of the trapped particle, and then apply a feedback loop able to decrease the position fluctuations. Due to the continuous nature of the measurement and to the effect of the trapping potential coupling the particle position with its momentum, feedback will realize an effective phase-space localization and cooling.

The proposed scheme needs a Doppler pre-cooling stage, as it happens for resolved sideband cooling. In fact, the effective trapped particle position measurement is realized only in the Lamb-Dicke regime, i.e. when the recoil energy is much smaller than the energy of a vibrational quantum, which is achieved only when the particle has undergone a preliminary cooling stage. Our scheme is able to provide further cooling, down to the motional ground state.

We shall consider the one-dimensional case, even if the method can be extended to the three-dimensional case. The trapped particle can be an ion in a rf-trap or a neutral atom in an optical trap. Our scheme however does not depend on the specific trapping method employed, and therefore we shall always refer to a generic trapped "atom", with frequency ν and annihilation operator a. The basic ingredient is the coupling with a driven cavity mode, with frequency ω_b and annihilation operator b, quasi-resonant with the transition between two internal atomic levels $|+\rangle$ and $|-\rangle$. In the frame rotating at the driving frequency ω_B, the system Hamiltonian is

$$H = \frac{\hbar\Delta}{2}\sigma_z + \hbar\nu a^\dagger a + \hbar\delta b^\dagger b + i\hbar\epsilon(\sigma_+ b - \sigma_- b^\dagger)\sin\left(kx + \phi\right) + i\hbar\left(\mathcal{B} b^\dagger - \mathcal{B}^* b\right) , \qquad (1)$$

where $\sigma_z = |+\rangle\langle+| - |-\rangle\langle-|$, $\sigma_\pm = |\pm\rangle\langle\mp|$, $\mathcal{B}$ is the amplitude of the driving laser, $\Delta = \omega_0 - \omega_B$ and $\delta = \omega_b - \omega_B$ are the atomic and field mode detuning, respectively.

An effective, continuous, measurement of the atom position can be obtained from an appropriate continuous homodyne measurement on the system. In fact, in the off-resonant case of large Δ, the excited level $|+\rangle$ can be adiabatically eliminated and, in the Lamb-Dicke limit, the homodyne photocurrent associated with the measurement of a cavity mode field quadrature becomes proportional to the real time dynamics of the atom position, $x(t)$. In the case when the cavity output light cannot be used, one could consider the resonant case $\omega_B = \omega_b = \omega_0$ and homodyne the atomic fluorescence: a continuous signal proportional to the atomic position is obtained also in this case.

The homodyne photocurrent is then fed back in order to displace the atom toward its equilibrium position so to decrease the position uncertainty. This means adding a feedback Hamiltonian linear in the atomic momentum p

$$H_{fb}(t) \propto I_{hom}(t) g p,$$

which can be realized with appropriate laser pulses. The resulting dynamics can be analyzed [3], and when the feedback gain g is chosen so to satisfy the stability condition, a stationary *thermal* state is achieved, with effective mean vibrational number N_{eff}. When optimization with respect to the feedback gain is performed, one gets an optimal effective mean excitation number

$$N_{eff} = \frac{1}{2}\left[\sqrt{\frac{1}{\eta}} - 1\right],$$

which is limited only by the efficiency of the homodyne measurement η. It is clear therefore that motional ground state cooling can be achieved when the homodyne measurement of the cavity mode is performed (off-resonant case), since in this case η can be very close to 100%. In the resonant case, when the effective atomic position measurement is achieved by homodyning the fluorescence, the efficiency is much lower and ground state cooling is much more difficult. Notice that the present cooling scheme works even in the presence of moderate motional heating processes [3], which can be caused by technical imperfections as the fluctuations of trap parameters due to ambient fluctuating electrical fields in the ion trap case, and due to laser intensity noise and beam-pointing fluctuations in the case of far-off resonance optical traps.

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